The CGC: An effective theory of QCD at high energies

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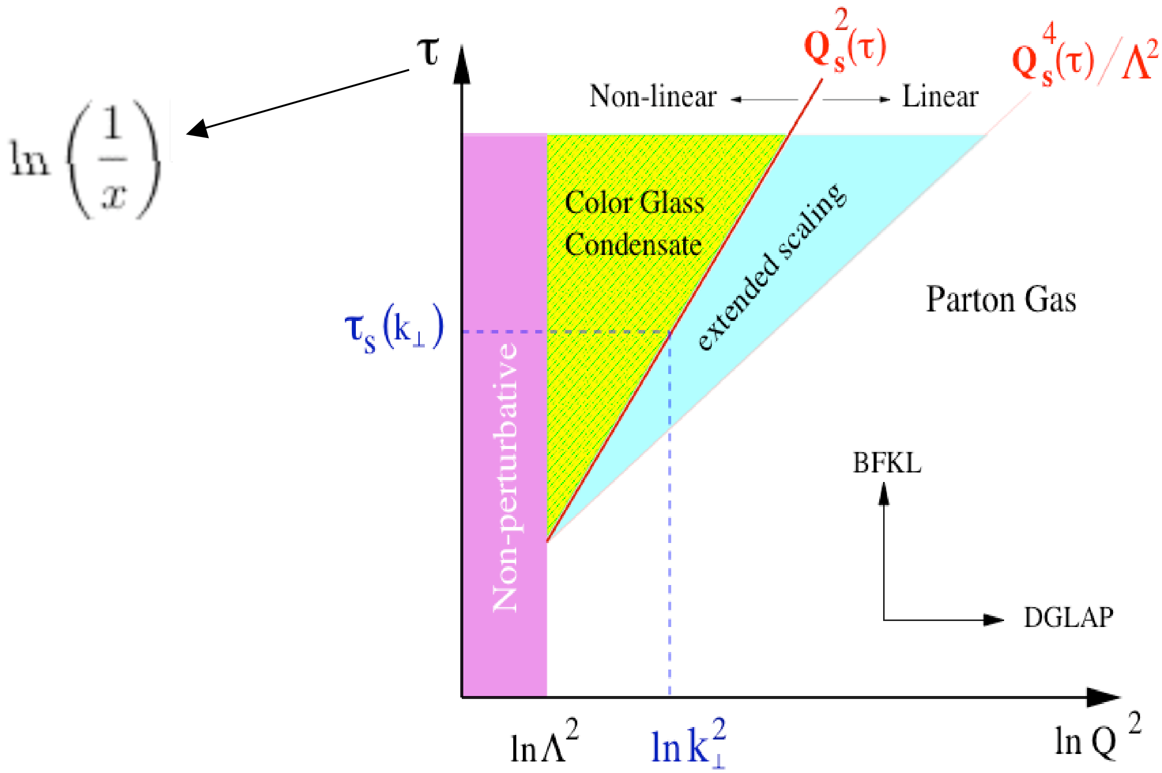
Outline of lectures

- Lecture I: *General introduction, the DIS paradigm, QCD evolution, saturation, the IMF hadron wave fn.*
- Lecture II: *The MV model, quantum evolution in the CGC, Wilson RG, analytic and numerical solutions.*
- Lecture III: *DIS and hadronic scattering at high energies*

The Color Glass Condensate: An effective field theory of QCD at high energies

- Life on the Light Cone
- The MV-model
- Quantum evolution: a Wilsonian RG
- The JIMWLK equations
- Analytical approximations and numerical solutions

Novel regime of QCD evolution at high energies

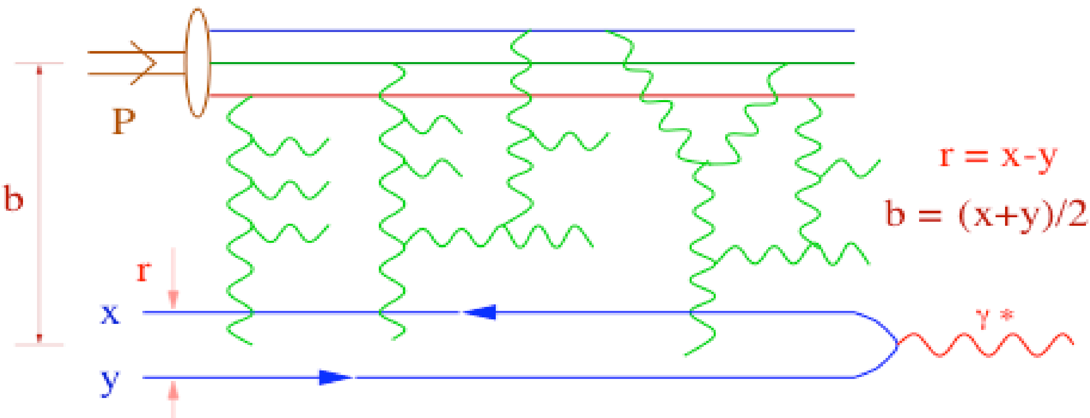


The Color Glass Condensate

The Balitsky-Kovchegov equation

I. Balitsky;
Y. Kovchegov

DIS:



$$\sigma^{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2 r |\psi(z, r; Q^2)|^2 \sigma_{\mathrm{dipole}}(x, r)$$

where $\sigma_{\mathrm{dipole}}(x, r) = 2 \int d^2 b \, (1 - S(x, r, b))$

McLerran,RV

with $S(x, r, b) = \frac{1}{N_c} < \mathrm{Tr} V^\dagger(x) V(y) >_Y \equiv 1 - \mathcal{N}_Y(r, b)$

s-matrix

amplitude

Path ordered exponential $V^\dagger(x) = \mathcal{P} \exp \left(ig \int dx^- \alpha_a(x^-, x) T^a \right)$

➢ Weak field limit: $V^\dagger(x) \approx 1 + ig\alpha(x)\ ;\ g\alpha << 1$

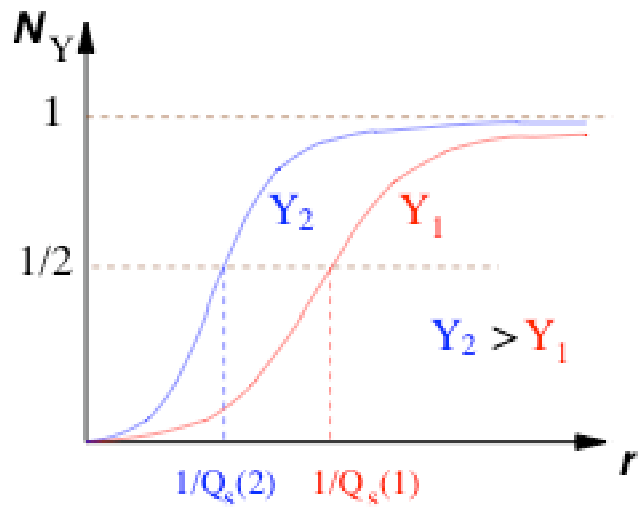
$$=> \mathcal{N}_Y(r) \sim \alpha_s r^2 \frac{xG(x, 1/r^2)}{\pi R^2}$$

violates unitarity bound if $\mathcal{N} > 1$

➢ For $r > 1/Q_s(Y)$ dipole probes strong fields $(g\alpha \sim 1)$

Iancu-McLerran RPA => $< V^\dagger(x) V(y) >_Y << 1 \text{ for } |x - y| >> 1/Q_s(Y)$

=> N ~ 1 - dipole unitarizes



Choose $\mathcal{N} = \frac{1}{2}$ as saturation condition to determine Q_s

BK: Evolution eqn. for the dipole cross-section

- The 2-point correlator $< V^\dagger(x) V(y) >$ in JIMWLK has

a closed form expression for $N_c \to \infty$ and $A >> 1$

$$\frac{\partial \mathcal{N}_Y(x,y)}{\partial Y} = \bar{\alpha}_s \int_z \frac{(x-y)^2}{(x-z)^2(y-z)^2} \{\underbrace{\mathcal{N}_Y(x,z) + \mathcal{N}_Y(z,y) - \mathcal{N}_Y(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{N}_Y(x,z)\mathcal{N}_Y(z,y)}_{\text{Non-linear}}\}$$

- For small dipole , $(r << 1/Q_s(Y)) => \mathrm{BFKL\ eqn.}$

$$\mathcal{N}_Y(r) \approx (r^2 Q_0^2)^{1/2} e^{\omega \bar{\alpha}_s Y} \exp\left(-\frac{\ln^2(1/r^2 Q_0^2)}{2\beta \bar{\alpha}_s Y}\right)$$

- From saturation condition,

$$\mathcal{N} = 1/2 \text{ when } r \sim 1/Q_s(Y) => \boxed{Q_s^2(Y) \approx Q_0^2\, e^{\lambda Y} \text{ with } \lambda \sim 4.8\, \alpha_s}$$

- For large dipole, $(r >> 1/Q_s(Y))$

$$\mathcal{N}_Y(r) \approx 1 - \kappa \exp\left(-\frac{1}{4c} \ln^2(r^2 Q_s^2(Y))\right)$$

$c \approx 4.8$

Levin,Tuchin;
Iancu,McLerran;Mueller

Numerical solutions of the BK-Eqn.

BFKL

BK

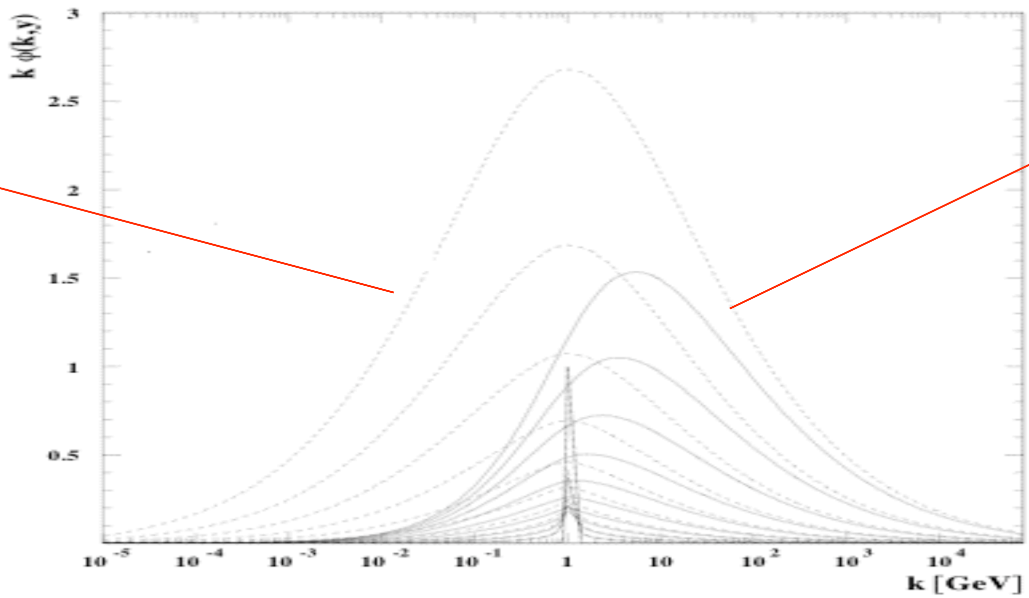


Figure 1: The functions $k\phi(k, Y)$ constructed from solutions to the BFKL and the Balitsky-Kovchegov equations for different values of the evolution parameter $Y = \ln(1/x)$ ranging from 1 to 10. The coupling constant $\alpha_s = 0.2$.

From K. Golec-Biernat, L. Motyka, A. M. Stasto, Phys Rev D65 (2002) 074037; hep-ph/0110325

No infrared diffusion a la BFKL in BK

Exact analogy to travelling waves => Munier, Peschanski

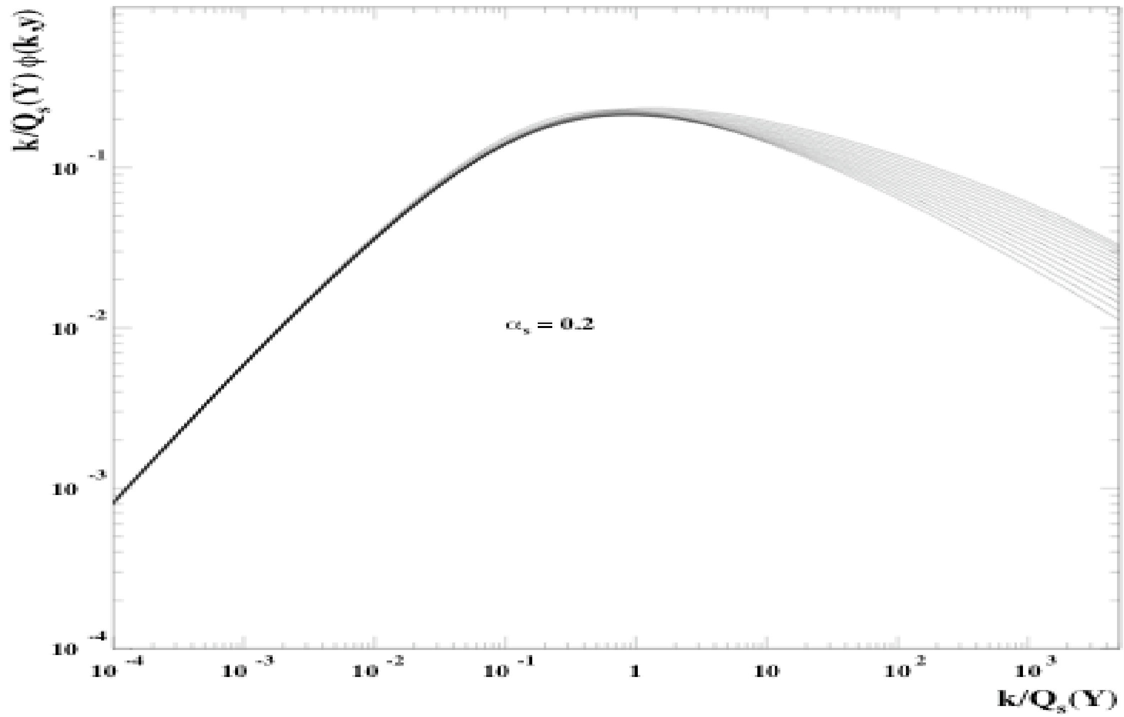


Figure 2: The function $\boxed{(k/Q_s(Y))\,\phi(k,Y)}$ plotted versus $\boxed{k/Q_s(Y)}$ for different values of rapidity Y ranging from 10 to 23. The saturation scale $Q_s(Y)$ corresponds to the position of the maximum of the function $k\,\phi(k,Y)$.

From K. Golec-Biernat, L. Motyka, A. M. Stasto, Phys Rev D65 (2002) 074037; hep-ph/0110325

Synopsis of CGC numerics

Numerical simulations of BK-eqn display Geometrical Scaling
(Armesto,Braun; Golec-Biernat,Stasto,Motyka)

Infrared diffusion pathology of BFKL is cured.

State of the art: numerical simulations of JIMWLK n-point correlators by Rummukainen & Weigert

Running coupling effects important & still to be understood…

Geometrical Scaling

Iancu,Itakura, McLerran; Mueller,Triantafyllopolous

Can write the solution of BFKL as:

$$\mathcal{N}_Y(r_\perp) \approx \exp\left(\omega\bar{\alpha}_s Y - \frac{\rho}{2} - \frac{\rho^2}{2\beta\bar{\alpha}_s Y}\right) \text{ with } \rho = \ln\frac{1}{r^2 Q_0^2}$$

ρ_S soln. where argument vanishes

$$\boxed{=> Q_s^2 = Q_0^2\, e^{c\bar{\alpha}_s Y}, \text{with } c = 4.84}$$

For $r_\perp < 1/Q_s$ (but close!), can write

$$\rho = \rho_S(Y) + \ln\frac{1}{r_\perp^2 Q_s^2} \equiv \rho_S + \delta\rho$$

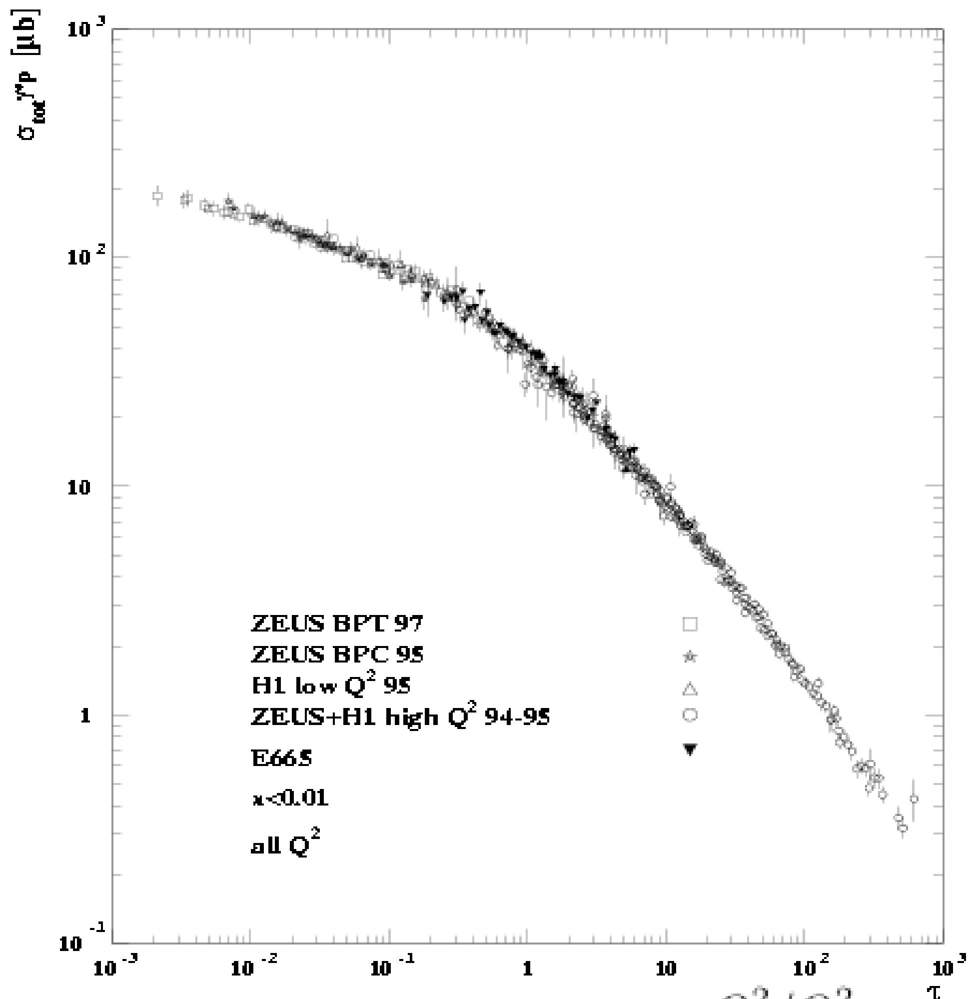
Plugging into N_Y, can show simply

$$\boxed{\mathcal{N}_Y \approx \left(r_\perp^2 Q_s^2(Y)\right)^\gamma \text{ for } Q_s^2 << Q^2 << \frac{Q_s^4}{Q_0^2}}$$

$\gamma \sim 0.64$ is large than BFKL anomalous dimension ~0.5

Geometrical scaling at HERA

(Golec-Biernat,Kwiecinski,Stasto)



$\tau = Q^2/Q_s^2$

Scaling seen for all x ‹ 0.01 and $0.045 < Q^2 < 450\,\mathrm{GeV}^2$

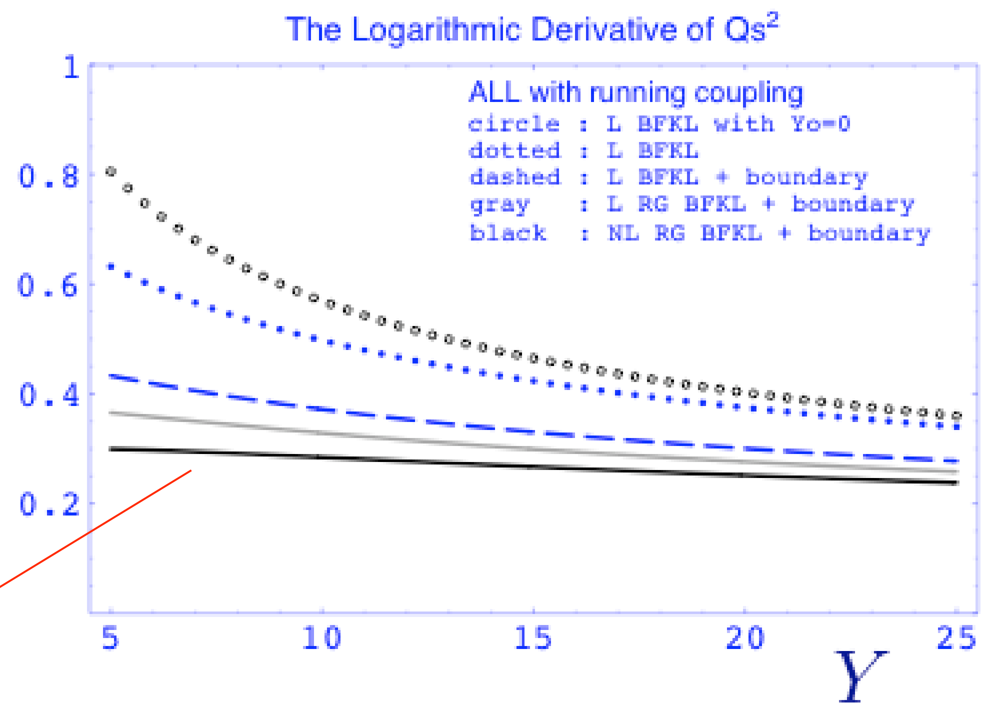
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\mathrm{QCD}}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

Re-summed NLO BFKL + CGC:

$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$

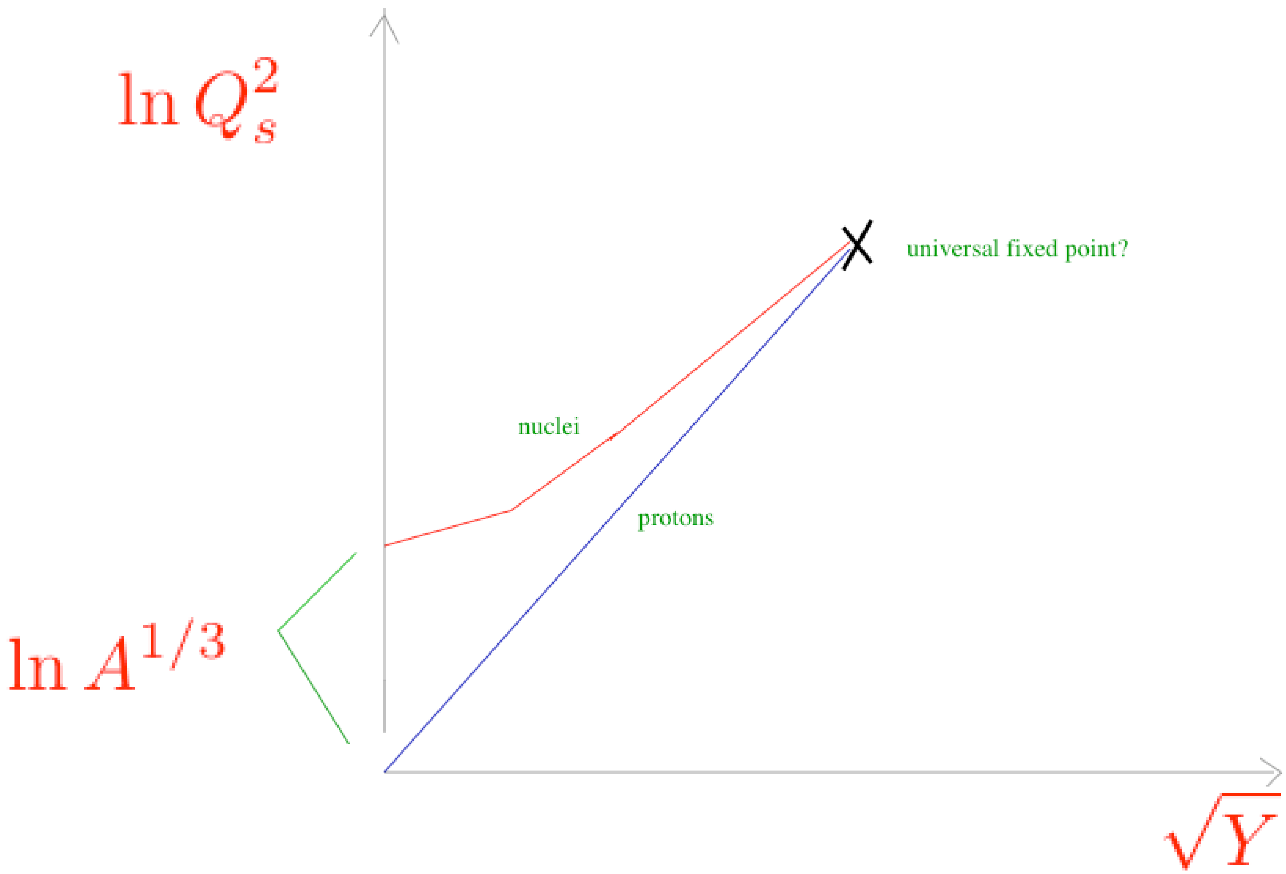


Triantafyllopolous

Very close to
HERA result!

A-dependence of saturation scale

Mueller



Such interesting systematics may be tested at LHC & eRHIC

Hadron & Nuclear Scattering at high energies

Introduction:

- Analytical & numerical studies of initial & final state effects in high energy hadronic scattering.

- Is “k_t factorization” of gluon & quark cross-sections a good assumption in p/D-A & AA-collisions?

- Relative importance of multiple scattering “Cronin” vs quantum evolution (geometrical scaling) effects on gluon and quark production in p/D-A and A-A collisions.
(see talk by Iancu)

- Initial conditions for Heavy Ion Collisions. Does the system thermalize ?

Systematic power counting for scattering in the CGC

Gluon & quark production to lowest order in sources (the dilute/pp case).

Gluon & quark production to lowest order in one source & all orders in the other (the semi-dense/pA case).

Gluon & quark production to all orders in both sources (the dense/AA case)

Dynamical evolution of soft & hard modes at late times in AA collisions

Gluon & quark production in the dilute/pp region

$(\rho_{p1}/k_\perp^2, \rho_{p2}/k_\perp^2) << 1$

Collinear Factorization:

Incoming partons have k_t=0. Applicable for $\boxed{Q \sim \sqrt{s} >> \Lambda_{\rm QCD}}$

Gluon & quark distributions evaluated at the scale Q^2 *Are universal*

K_t factorization:

Collins & Ellis; Catani, Ciafaloni & Hautmann

Incoming partons have k_t-applicable when $\boxed{\Lambda_{\rm QCD} << Q << \sqrt{s}}$

Described by unintegrated parton dists. $\phi_{p,A}(k_\perp)$

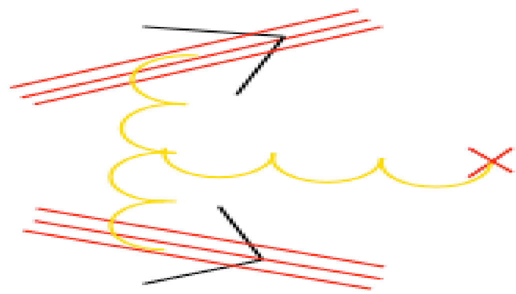
Is this k_t scale the saturation scale k_t ~ Q_s ? Levin, Ryskin, Shabelski, Shuvaev

Several phenomenological studies by LRSS and Hagler et al studying spectra and correlations in pp-collisions

(Related approach by Raufeisen, Kopeliovich, Tarasov)

CGC is powerful formalism to study these issues at high energies. Collinear and k_t factorization arise as specific limits of the formalism

- Inclusive gluon production in hadronic collisions to lowest order in ρ^1, ρ^2 and in α_S expressed in k_t factorized form



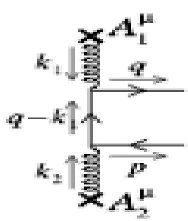
Kovner, McLerran, Weigert
Kovchegov, Rischke
Gyulassy, McLerran

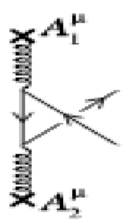
This diagram in $A^\tau = 0$ gauge is equivalent to sum of all bremsstrahlung diagrams in covariant gauge

- **Inclusive pair-production in CGC framework**

Gelis, RV

Work in $\partial_\mu A^\mu = 0$ gauge





Abelian

$A^\mu_{12} \propto O(\rho 1 \rho 2)$

non-Abelian-vertex here is the Lipatov vertex C^μ

$$\frac{d\sigma}{dy_p dy_q d^2p_\perp d^2q_\perp} = \frac{1}{(2\pi)^6}\frac{1}{(N_c^2-1)^2}\int \frac{d^2k_{1\perp}}{(2\pi)^2}\frac{d^2k_{2\perp}^2}{(2\pi)^2}\delta(\vec{k}_{1\perp}+\vec{k}_{2\perp}-\vec{p}_\perp-\vec{q}_\perp)$$
$$\times\phi_1(k_{1\perp})\phi_2(k_{2\perp})\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

$|m_{ab}^{-+}(k_1,k_2;q,p)|^2$ *is identical to Collins & Ellis' k_t factorization result*

$$\frac{d\phi_1(k_{1\perp},x_\perp)}{d^2x_\perp} = \frac{\pi g^2}{k_\perp^2}\int d^2r_\perp e^{-i\vec{k}_\perp\cdot\vec{r}_\perp} < \rho_a(x_\perp+\frac{r_\perp}{2})\rho_a(x_\perp-\frac{r_\perp}{2}) >_\rho$$

is the un-integrated gluon distribution in the Gaussian MV-

$$\frac{\mathrm{Tr}\left(|m_{ab}^{-+}(k_1,k_2;q,p)|^2\right)}{k_{1\perp}^2 k_{2\perp}^2}$$

is well defined in the collinear limit of $|k_{1\perp}|, |k_{2\perp}| \to 0$

$\downarrow$

$|M|^2_{gg\to q\bar{q}}$ *after integration over azimuthal angles*

Recover lowest order collinear factorization result

Gluon & quark production in the semi-dense/pA region

$\left(\rho_p/k_\perp^2 << 1, \rho_A/k_\perp^2 \sim 1\right)$

Blaizot, Gelis, RV

- Solve classical Yang–Mills eqns.

$$[D_\mu, F^{\mu\nu}] = J^\nu \; ; \; [D_\nu, J^\nu] = 0$$

with two light cone sources

$$J^{\nu,a} = \underbrace{\delta^{\nu +}\delta(x^-)\rho_1^a(x_\perp)}_{\text{proton source}} + \underbrace{\delta^{\nu -}\delta(x^+)\rho_2^a(x_\perp)}_{\text{nuclear source}}$$

- $\boxed{\partial_\mu A^\mu = 0}$ => equations can be written as

$$(2\partial^+\partial^- - \nabla_\perp^2)A^\nu = J^\nu + ig\,[A_\mu, F^{\mu\nu} + \partial^\mu A^\nu]$$

need $A^\mu_{1\infty}$ =order $O(\rho_1)$ in proton & order $O(\rho_2^n)$; $n \to \infty$ in nucleus

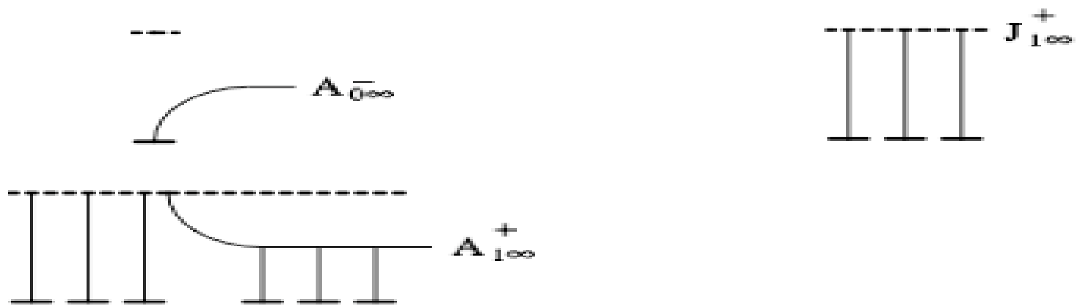
$$\begin{aligned}
(\partial^- + igA^-_{0\infty} \cdot T)J^+_{1\infty} &= 0 \\
(2\partial^+\partial^- - \nabla_\perp^2 + igA^-_{0\infty} \cdot T\partial^+)A^+_{1\infty} &= J^+_{1\infty} \\
(2\partial^+\partial^- - \nabla_\perp^2 + 2igA)0\infty^- \cdot T\partial^+)A^i_{1\infty} &= ig(A^-_{0\infty} \cdot T)\partial^i A^+_{1\infty} - ig(\partial^i A^-_{0\infty} \cdot T)A^+_{1\infty} \\
A^-_{1\infty} &= \frac{1}{\partial +}(\partial^i A^i_{1\infty} + \partial^- A^+_{1\infty})
\end{aligned}$$

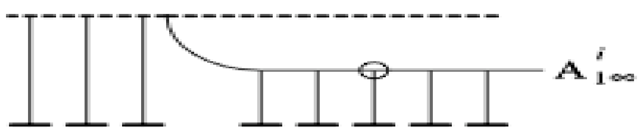
$$A^-_{0\infty} = -\delta(x^+)\frac{1}{\nabla_\perp^2}\rho_2(x_\perp) \qquad J^+_{1\infty} \to A^+_{1\infty} \to A^i_{1\infty} \to A^-_{1\infty}$$

Diagrammatic Representation

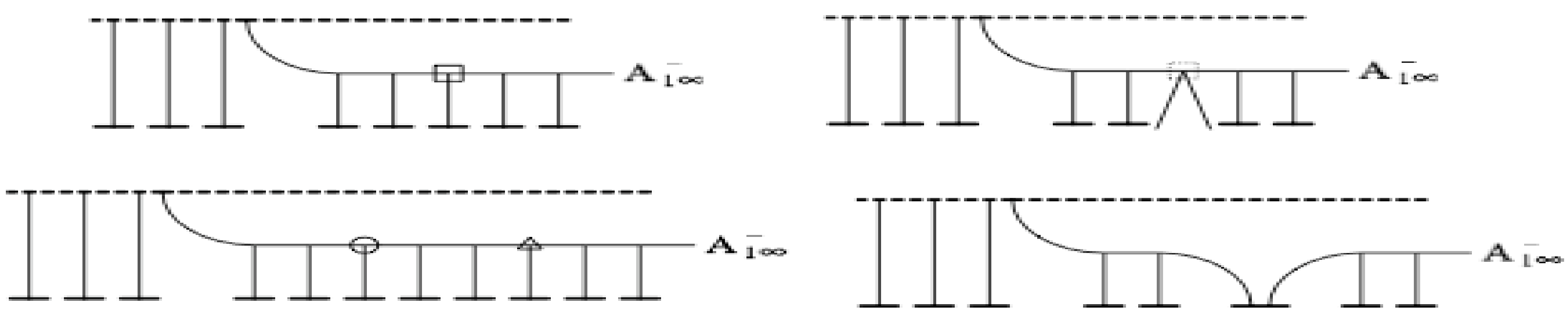
Solid line = 1 power of ρ_2

Dashed line=1 power of ρ_1





Circular vertex is the vertex Γ^{i-+} for the transition $A^+_{1\infty} \to A^i_{1\infty}$



- The field $A^-_{1\infty}$ can be computed from the gauge condition $\partial_\mu A^\mu = 0$

- The gluon field produced in pA collisions has the compact form:

$$q^2 \tilde{A}^\mu_{1\infty}(q) = i \int \frac{d^4k}{(2\pi)^4} \left(C^\mu_U U(k_2) + C^\mu_V V(k_2) + C^\mu_1 \mathbf{1}(k_2)\right) 2\pi\delta(k^-) \frac{\rho_1(k_\perp)}{k_\perp^2}$$

$$\mathrm{F.T.}\mathcal{P}_+ \exp\left[ig \int_{-\infty}^{\infty} dz^+ A^-_A(z^+, y_\perp)\cdot T\right] \qquad \mathrm{F.T.}\mathcal{P}_+ \exp\left[\frac{ig}{2} \int_{-\infty}^{\infty} dz^+ A^-_A(z^+, y_\perp)\cdot T\right]$$

- The well known Lipatov vertex is simply

$$C^\mu_L = C^\mu_U + \frac{1}{2} C^\mu_V$$

For on-shell gluons,

$$\boxed{C^\mu_1 = 0\,;\, C_U \cdot C_V = C_V^2 = 0 \text{ and } C_U^2 = C_V^2 = -\frac{4k_{1\perp}^2 k_{2\perp}^2}{q_\perp^2}}$$

Thus only bi-linears of Wilson line U survive in the squared amplitude

- Final result for the gluon multiplicity in pA

$$\boxed{N_g = \frac{4g^2 N_c}{\pi^2 (N_c^2 - 1) q_\perp^2} \int \frac{d^3 q}{(2\pi)^3 2E_q} \frac{d^2 k_\perp}{(2\pi)^2} \int d^2 x_\perp \frac{d\phi_p(k_\perp, x_\perp)}{d^2 X_\perp} \frac{d\phi_A(q_\perp - k_\perp, x_\perp - b)}{d^2 X_\perp}}$$

k_t factorized into product of proton * nuclear unintegrated distributions

Kovchegov, Mueller
Kovchegov,Tuchin
Kovchegov,Kharzeev,Tuchin

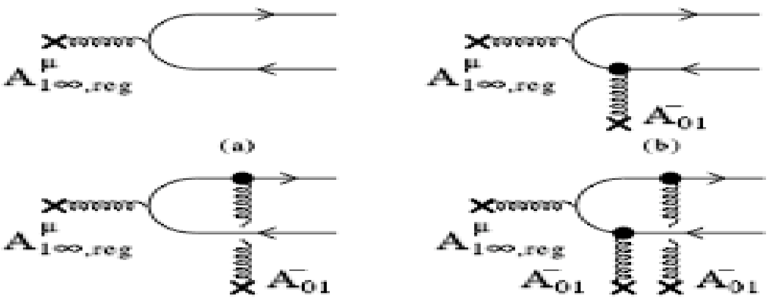
$\phi_A(k_t, x_\perp) \propto < U^\dagger_{ab} U_{bc} >_{\rho^2}$ —is non—linear—contains gluon density to all orders—proportional to gluon density at large k_t

- Exactly equivalent to result of Dumitru & Mclerran in $A^\tau = 0$ gauge

Dumitru,Jalilian—Marian,Gelis

- Cronin effect? See Iancu's talk

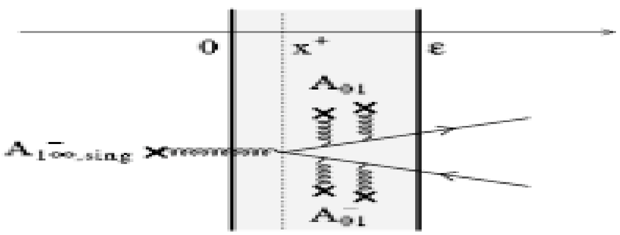
Quark production to all orders in pA



$A^{\mu}_{1\infty}$ is the gluon field to $O(\rho_1 \rho_2^n)$ $n \to \infty$

- Computed both Feynman & retarded amplitudes— differ only by a phase.

- Again, the V–Wilson lines disappear—need contribution from pair scattering in nucleus



- Result for neither quark pair production nor single quark production is k_t factorizable

Blaizot,Gelis,RV

Result can however still be factorized

$$\frac{d\sigma^{pA\to q\bar{q}X}}{dy_p dy_A d^2p_\perp q_\perp} \propto \phi_p \times [A\phi_{g,g} + (B\phi_{g;q\bar{q}} + c.c) + C\phi_{q\bar{q};q\bar{q}}]$$

$$< U_A(x_\perp)U_A^\dagger(y_\perp) >$$

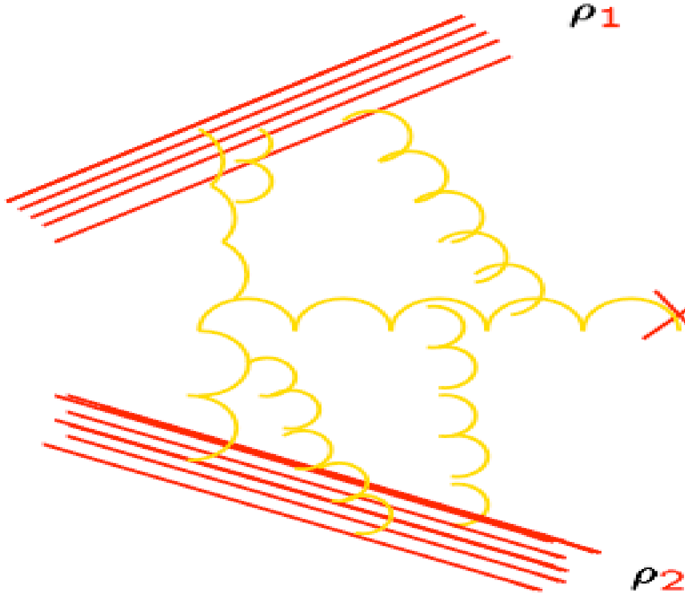
$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)U_F(y'_\perp)\tau^b U_F(x'_\perp) >$$

$$< U_F(x_\perp)\tau^a U_F^\dagger(y_\perp)\tau^{b'}(U_A^{a'b'})^\dagger(y'_\perp) >$$

These correlators can be computed with JIMWLK RG equations.

Gluon & Quark production in the dense/AA region

$(\rho_1/k_t^2, \rho_2/k_t^2 \sim 1)$



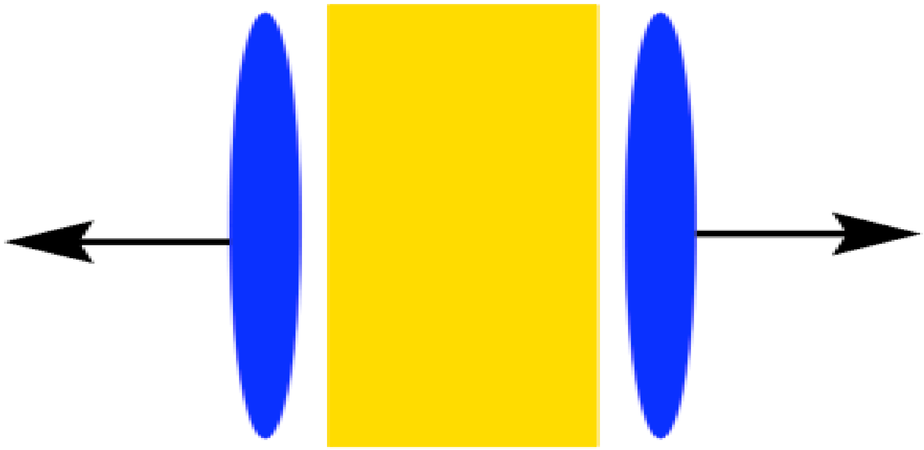
- Likely not k_t factorizable—only solved numerically thus far

  Krasnitz,RV
  Krasnitz,Nara,RV
  Lappi

- Wave—fn evolution effects difficult to include —work of Rummukainen & Weigert promising...

Quantum evolution included only through saturation scale in KNV

Real Time Gluodynamics of Nuclear Collisions



Kovner,McLerran,Weigert
Krasnitz,Nara,Venugopalan
Lappi

Classical Fields with occupation # $f = \frac{1}{\alpha_s}$

- Non−perturbative formulae for initial glue distributions

$$\frac{1}{\pi R^2}\frac{dE_T^{\rm glue}}{d\eta} = \frac{0.25}{g^2} Q_s^3$$

$$\frac{1}{\pi R^2}\frac{dN^{\rm glue}}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

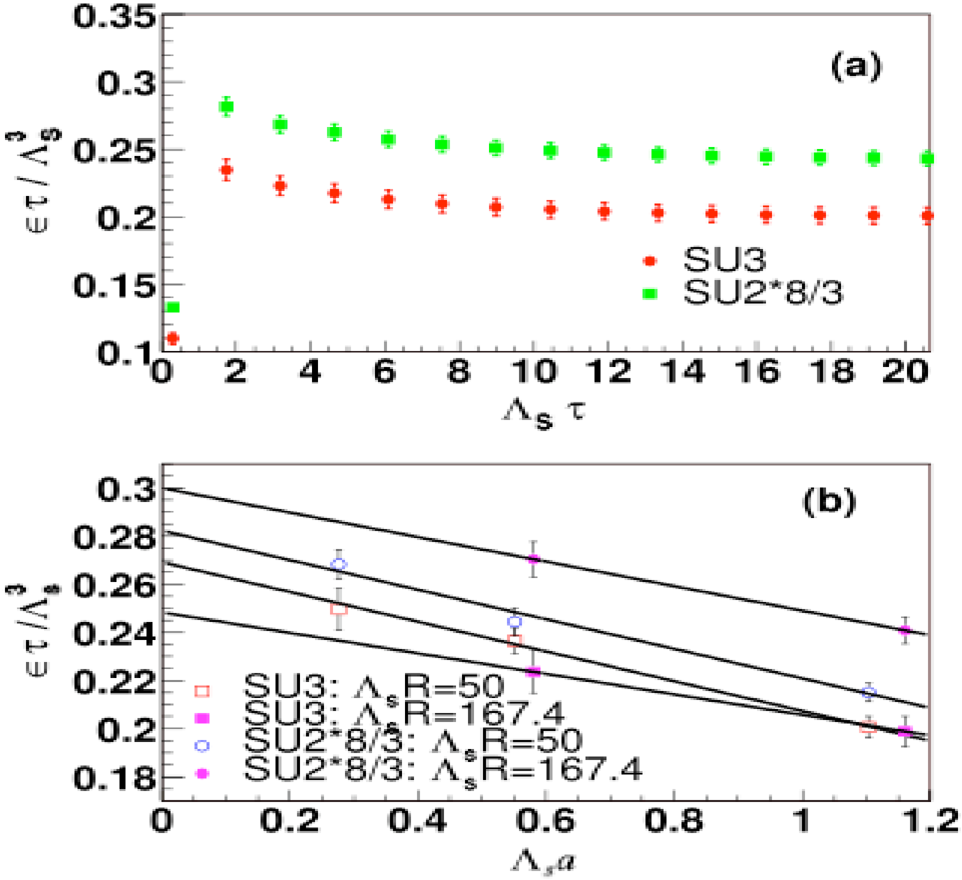
- Classical approach breaks down at late time when $f << 1$...

$\tau >> \frac{1}{Q_s}$ but $\tau << R$

Results

Total energy of gluons

$$\frac{1}{\pi R^2}\frac{dE_\perp}{d\eta}\Big|_{\eta=0} = \frac{f_E(\Lambda_s R)}{g^2}\Lambda_s^3$$



Proper time dependence:

$$\varepsilon\tau = \alpha + \beta\exp(-\gamma\tau)$$

$dE_\perp/d\eta/\pi R^2 = \alpha$ is the energy density and

$$\tau_D = 1/\gamma/\Lambda_s$$

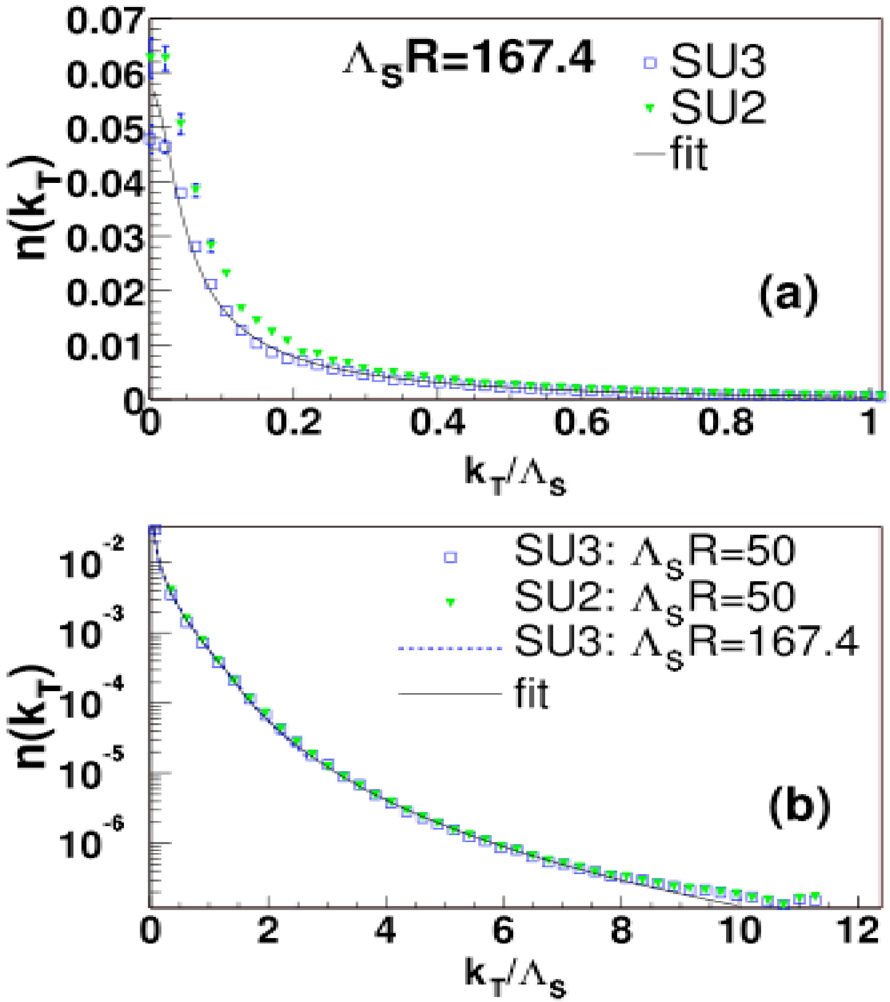
is the "formation time"

(~0.3 fm for RHIC and ~0.1 fm for LHC)

The energy density at τ_D *is then*

$$\varepsilon = \frac{0.08}{g^2}\Lambda_s^4$$

Transverse momentum distributions of gluons



$$n(k_\perp) = \tilde{f}_N/(N_c^2-1)$$

The SU(3) gluon distribution is fitted by the form

$$\frac{1}{\pi R^2}\frac{dN}{d\eta d^2k_\perp} = \frac{\tilde{f}_N}{g^2}$$

where

$$\tilde{f}_N = \frac{a_1}{\exp\left(\frac{\sqrt{k_\perp^2+m^2}}{T_{\rm eff}}\right)-1}$$

for $k_\perp/\Lambda_s < 1.5$

and

$$\tilde{f}_N = a_2\,\Lambda_s^4 \ln(4\pi k_\perp/\Lambda_s)\, k_\perp^{-4}$$

for $k_\perp/\Lambda_s > 1.5$

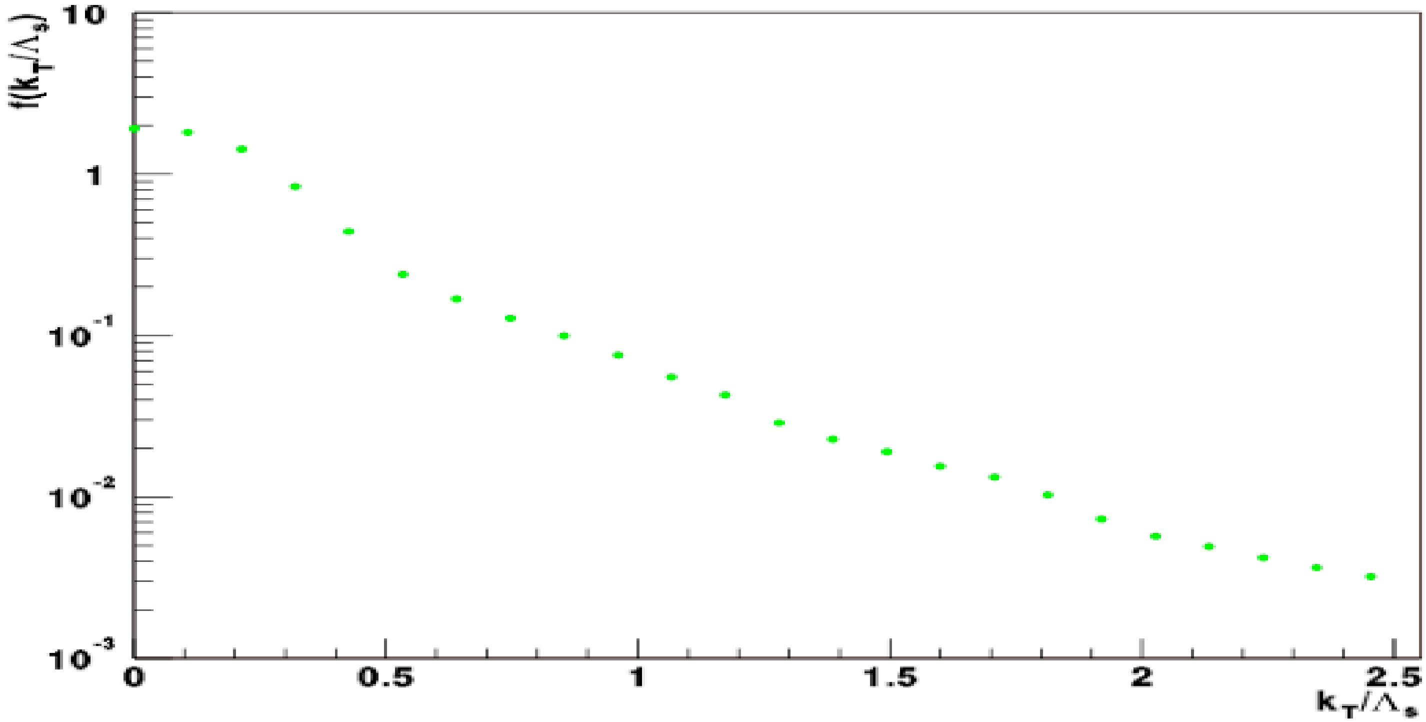
$$T_{\rm eff} = 0.47\Lambda_s$$

$$a_1 = 0.137\ ;\ a_2 = 0.009\ ;\ m = 0.04\Lambda_s$$

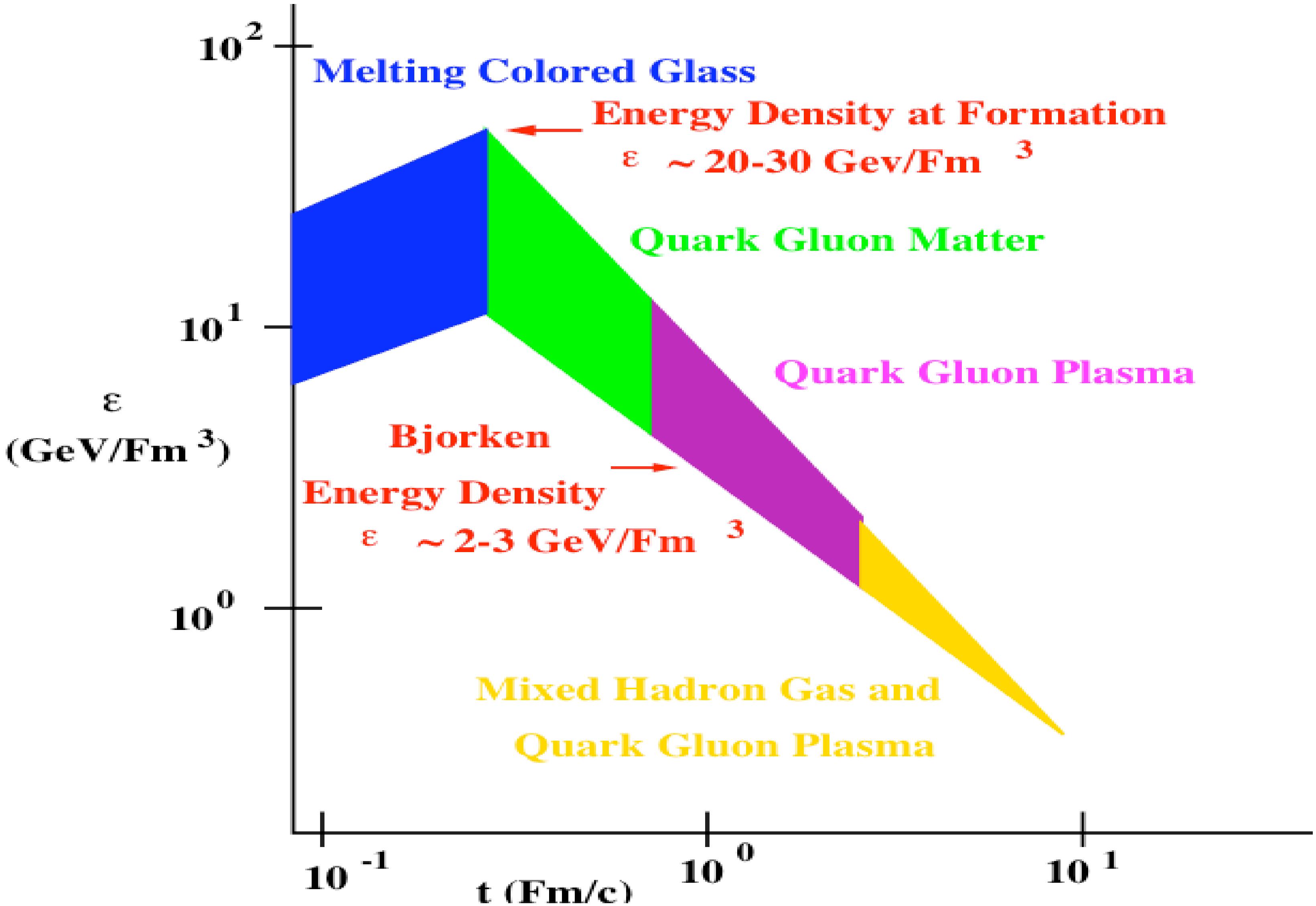
- *The transverse momentum dist. is infrared finite...*

Occupation #

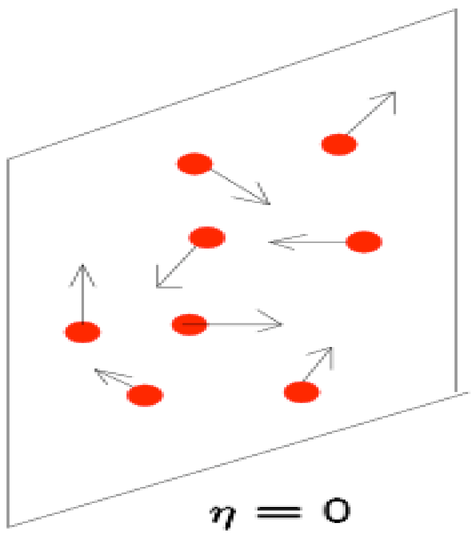
$$f = \frac{(2\pi)^3}{2(N_c^2 - 1)} \frac{dN}{d^3x d^3p}$$



Space-time history of a heavy ion collision



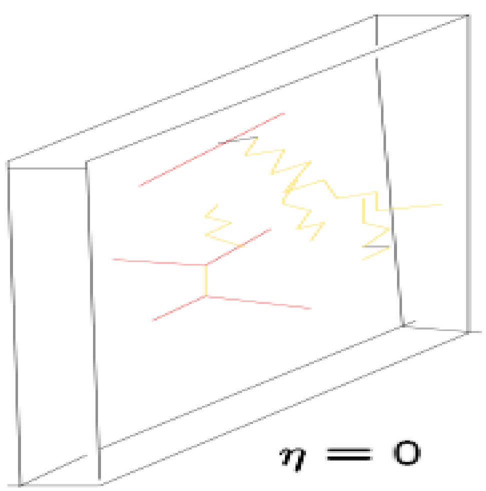
- *The CGC describes only the initial state—produced gluons may re—scatter and thermalize...*



$\tau \sim 1/\Lambda_s$

$p_\perp \sim \Lambda_s$

$p_z \sim 0$

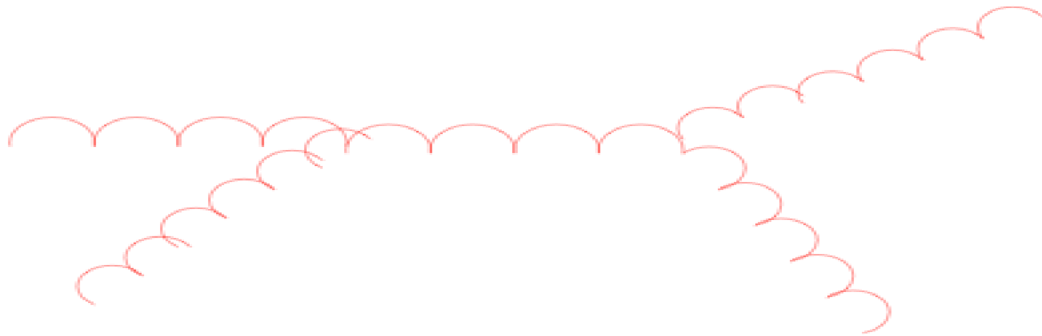


$1/\Lambda_s << \tau << R$

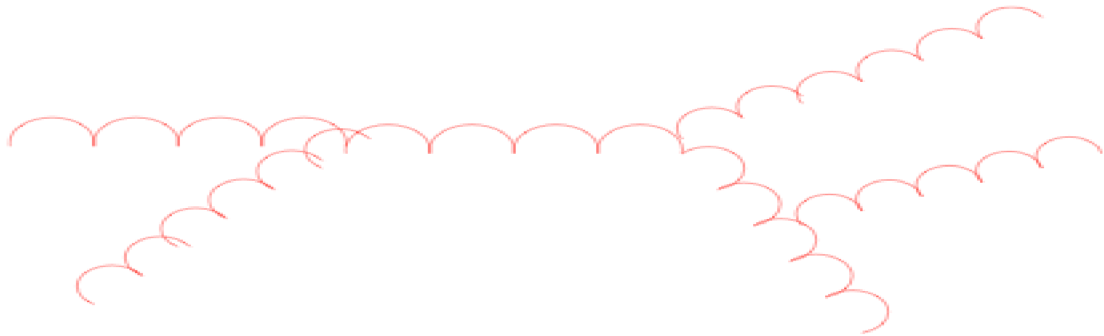
$p_\perp \sim p_z \sim T$

A. H. Mueller
Bjoraker , R.V

- *Small angle scattering drives the system only slowly towards equilibrium...*



- *2 ---> 3 processes may be more efficient...*



Baier, Mueller, Schiff, Son

Role of collective instabilities in thermalization?

Arnold,Lenaghan,Moore

Outlook

Need self-consistent treatment of
soft & hard modes with CGC initial
conditions.

Early thermalization still a puzzle
In pQCD based approaches.